007. THERMAL OVERLOAD.

Is the carrying capacity of thermals limited?

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You have probably experienced this:

You are on a flight in really weak conditions and struggling to stay aloft. You manage to find a small thermal. You relax, bank steep, and patiently turn your circles moving slowly upwards. You are happy to survive for a bit longer.

All the sudden a desperate large gaggle appears. They spot you, and franticly join you in your climb. Because of the sheer amount of plastic in the air, combined with nervousness, you are hindered in your path and out of necessity you open up your circles while evading other gliders. As a result, your already small climb rate drops to zero.

A big chaotic carrousel is turning in the air. And no one is climbing.

Suddenly, an agitated pilot understands the pointlessness of this all, and leaves the thermal. He is immediately followed by the rest of the flock. You stay behind out of frustration, now free again to fly as you like, and lo and behold: your climb rates are restored.

I always asked myself: is it strictly the enforced wide circles around the core, the constant shifting, and the inefficient steering maneuvers that destroy the climbrates? Or is it possible that the mass of all these gliders has a negative effect as well?

Could the mass of the gliders in a thermal influence the climb rate?

The Model

I don’t think it is helping anyone to answer this question in excruciating detail. Therefore, I constructed a spherical cow model[1] of a thermal, which I deem to be sufficient to get some rough answers. You can find the details of the model in the appendix.

The main thing to look for, is if the weight of the gliders is significant compared to the net buoyancy force of the thermal. One of the first things you learn in engineering, is to be careful when you subtract 2 nearly equal numbers from eachother. It makes the results very sensitive to small deviations in values, and thus unreliable. That’s what happening here. And thus, take these results as a back of a napkin sketch of what is going on, and not as highly accurate results.

The Results

We look at 5 different amounts and weights of gliders in the thermals:

- 0kg: Without Gliders
- 300kg: Single Glider 300kg
- 2,250kg: 5 Gliders 450 kg (small group standard class / club class gliders)
- 12,000kg: 20 Gliders 600 kg (flock of 18m gliders)
- 30,000kg: 40 Gliders 750 kg (large gaggle of mix of 18m, 20m or open class gaggle)

And for 5 different types of thermals:

- 0.7m/s climbrate, 750m height (125m radius, 0.370C Temperature difference)
- 1.0m/s climbrate, 1000m height, (150m radius, 0.445C Temperature difference)
- 1.6m/s climbrate, 1250m height, (175m radius, 0.675C Temperature difference)
- 1.8m/s climbrate, 1500m height, (175m radius, 0.795C Temperature difference)
- 2.5m/s climbrate, 1750m height, (200m radius, 1.120C Temperature difference)

Climb Rates (m/s)

		Thermal Height	750m	1000m	1250m	1500m	1750m
0	**kg**	**No Gliders**	**0.70**	**1.00**	**1.60**	**1.80**	**2.50**
300	kg	Single Glider	0.70	1.00	1.60	1.80	2.50
2,250	kg	5 Gliders	0.68	0.99	1.59	1.79	2.49
12,000	kg	20 Gliders	0.57	0.92	1.55	1.77	2.48
30,000	kg	40 Gliders	0.34	0.79	1.49	1.72	2.45

The results show, that for low ceilings and weak climb rates (eg 0.7ms/, 750m), it indeed seems plausible that the vertical speeds of thermals can be affected by the mass of the gliders inside. For just a few gliders, there will not be a noticeable effect. For very large gaggles, it should be very noticeable.

For any “average” or good thermal, the effect should be unnoticeable.

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[1] *Milk production at a dairy farm was low, so the farmer wrote to the local university, asking for help from academia. A multidisciplinary team of professors was assembled, headed by a theoretical physicist, and two weeks of intensive on-site investigation took place. The scholars then returned to the university, notebooks crammed with data, where the task of writing the report was left to the team leader. Shortly thereafter the physicist returned to the farm, saying to the farmer, "I have the solution, but it works only in the case of spherical cows in a vacuum".*

We can also think about this differently: How large is the weight of the gliders, compared to the net buoyance force of the thermal. If that is a significant amount, then it would be very likely that this negatively influences the climb rates.

% of glider weight to net buoyancyforce							
		Thermal Height	750m	1000m	1250m	1500m	1750m
300	kg	Single Glider	0.4%	0.2%	0.1%	0.1%	0.0%
2,250	kg	5 Gliders	3.2%	1.7%	0.7%	0.5%	0.2%
12,000	kg	20 Gliders	17.2%	9.0%	3.5%	2.5%	1.2%
30,000	kg	40 Gliders	43.0%	22.5%	8.7%	6.2%	2.9%

Indeed, we see that, for low ceilings and weak climb rates, the weight of large gaggles becomes significant compared to the Archimedes effect. In the case of 40 gliders in a 750m, 0.7m/s, the weight of the glider amounts to 43% of the net buoyancy force of the thermal separately.

Sensitivity Analysis

In the simple model, the climb rates of thermals strictly depend on the radius of the thermal, and the temperature difference between thermal and atmosphere. There are thus an infinite amount of combinations to obtain a single climb rate. Of course, those should be in a realistic interval.

But it thus needs to be checked, how the predictions of the model depend on those parameters. Below you'll find the results for a weak and average climb.

Climb Rates (m/s) - 750m - 0.7m/s							
		Temperature Difference (C)	0.46	0.37	0.31	0.265	0.23
		Thermal Radius (m)	100	125	150	175	200
0	**kg**	**No Gliders**	**0.70**	**0.70**	**0.70**	**0.70**	**0.70**
300	kg	Single Glider	0.69	0.70	0.70	0.70	0.69
2,250	kg	5 Gliders	0.66	0.67	0.68	0.68	0.68
12,000	kg	20 Gliders	0.49	0.54	0.57	0.59	0.60
30,000	kg	40 Gliders	0.08	0.24	0.34	0.39	0.43

Climb Rates (m/s) - 1250m - 1.6m/s							
		Temperature Difference (C)	0.95	0.79	0.675	0.59	0.525
		Thermal Radius (m)	125	150	175	200	225
0	**kg**	**No Gliders**	**1.60**	**1.60**	**1.60**	**1.60**	**1.60**
300	kg	Single Glider	1.60	1.60	1.60	1.59	1.60
2,250	kg	5 Gliders	1.59	1.59	1.59	1.59	1.59
12,000	kg	20 Gliders	1.54	1.55	1.55	1.56	1.56
30,000	kg	40 Gliders	1.45	1.47	1.49	1.50	1.51

So, for the weak climb, the results are sensitive to model parameters. But still, for the large gaggles, the results remain noticeable in all parameter settings.

For average climb rates (and upwards), the model is less sensitive. Climb rates are thus indeed likely not as impacted by the mass of the gliders in the thermal, even for big gaggles.

Conclusion

So, next time you find yourself in a situation similar to the one in the opening paragraph, remember it's not only the chaotic flying of the gliders in the gaggle that reduces your climb rate, it's plausibly also the weight of all the gliders. And there is little anyone can do about it.

Appendix: An overly simple thermal model

For the purpose of finding a ballpark effect, we don't need to make use of a complex model (Plume/jet models, parcel models, Lenschow, etc). The extremely simple method below will probably horrify some meteorologists. I need to stress that it is way too simple and inaccurate for most other applications.

The overly simple model of thermals:

- The atmosphere is static (it doesn't move, and is not influenced by the thermal), and air density, pressure and temperature are the same at all altitudes, and no moisture in the air.
- The thermal is a fixed steady cylinder, which is completely uniform and time-independent.
    - That means, fixed radius, fixed density, pressure, temperature, and fixed upward velocity at all locations in that cylinder.
    - The bottom of the cylinder is at the top of the superadiabatic layer (for arguments sake at 1m above the ground), and warm air flows in there.
    - The top of the cylinder is at the inversion or cloud base, and the air from the thermal flows out there.
    - Velocity (opposite sign of course), flow rate (mass and volume), density, temperature, pressure, are all the same at that bottom and top. This means, the net force from top and bottom of the cylinder is zero, and so these forces are ignored.
    - Mass $m_{thermal}$ of the cylinder is pulled down by gravity. The force equals the Weight

      $$W = \rho_{thermal}\, Vg = \rho_{thermal}\, \pi\, r^2\, hg$$

      With volume of the cylinder V, radius r, height h, and density of the thermal $\rho_{thermal}$
    - At the cylinder wall, there is a friction force D between the thermal and the cylinder sidewall, which scales quadratically with the vertical speed of the thermal.

      $$D = \frac{1}{2} C_d\, \rho_{air}\, S\, v^2 = C_d\, \rho_{air}\, \pi\, r\, h\, v^2$$

      With S the surface area of the cylinder sidewall. C_d is the drag coefficient, that some literature puts around 0.7 for thermals. ρ_{air} is the standard density of the atmosphere, 1.225kg/m3.

- The only upward force is the buoyance force B, which is equal to the displaced weight of the air with the volume of the thermal cylinder:

$$B = \rho_{air}\, Vg = \rho_{air}\, \pi\, r^2\, hg$$

- This of course a very simplified model, but the net buoyancy force and the sidewall friction of the thermal against the atmosphere are the major things at work in a real thermal as well, and thus the main mechanisms that determine the magnitude of the climb rate.

  The force balance of the three only forces in the model equals:

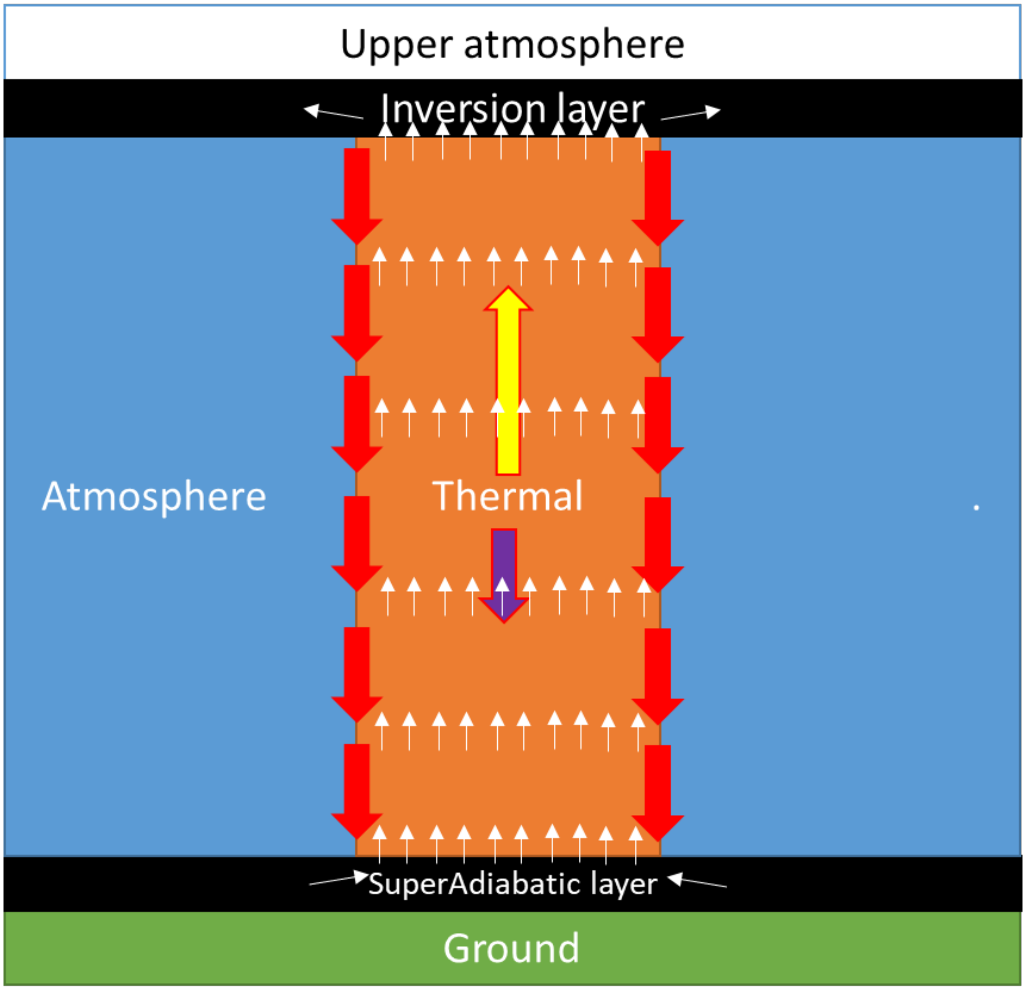
$$B - W = D$$

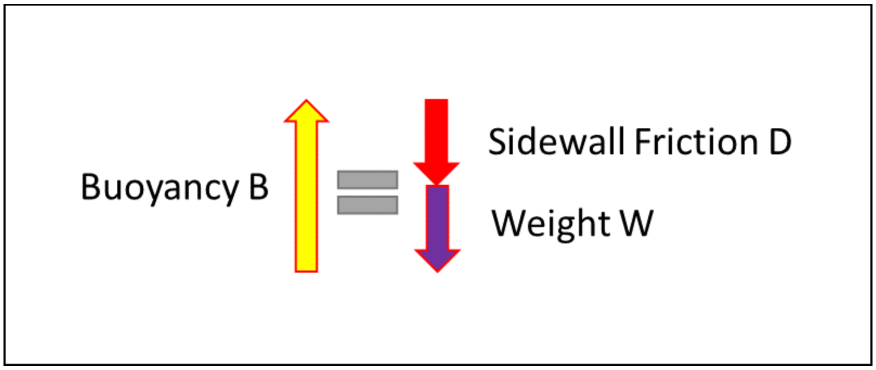
$$V\,(\rho_{air} - \rho_{thermal})g = \frac{1}{2} C_d\, \rho_{air}\, S\, v^2$$

$$\pi\, r^2\, h\,(\rho_{air} - \rho_{thermal})g = C_d\, \rho_{air}\, \pi\, r\, h\, v^2$$

$$v = \sqrt{\frac{rg}{C_d \rho_{air}} (\rho_{air} - \rho_{thermal})}$$

- In this model, vertical velocity of the thermals are thus completely described by radius of the thermal and density of the thermal.





Air density and thermal density are also only dependent on their respective temperatures in this model (no moisture).

$$\rho_{air} = \frac{p_{air}}{R\, T_{air}}$$

$$\rho_{thermal} = \frac{p_{air}}{R\, T_{thermal}}$$

With p_{air} fixed at the standard air pressure of 1013.25hPa, R the specific gas constant for dry air 287.05J/(kg K), and T_{air} and $T_{thermal}$ the temperatures of the atmosphere and thermal respectively. We also fix T_{air} at standard temperature of 15°C (288.15K).

$$v = \sqrt{\frac{rg}{C_d T_{air}} \left(T_{air} - T_{thermal}\right)}$$

$$v = \sqrt{\frac{r\, 9.83}{0.7 * 288.15} \left(288.15 - T_{thermal}\right)}$$

$$v = \sqrt{0.04873\, r\, (288.15 - T_{thermal})}$$

There is thus an unlimited amount of combinations of thermal radius r with thermal temperature that will lead to the same vertical velocity $T_{thermal}$. Since both values are estimates, we need to do a sensitivity analysis based on a realistic range of those values.

As a next step, we introduce the gliders in the thermal. We add up the mass of the gliders $m_{gliders}$ with the mass $m_{thermal}$ of the thermal, so we get m_{t+g}. And $W_{gliders}$ plus W equals W_{t+g}. The volume of the gliders is really small compared to the volume of the thermal, and are thus ignored.

A new ρ_{t+g} is introduced, which is the weight of the thermal and the weight of the gliders divided by V, the volume of the thermal

$$\rho_{t+g} = \frac{m_{t+g}}{V} = \frac{m_{gliders} + m_{thermal}}{V} = \frac{m_{gliders}}{V} + \rho_{thermal}$$

The force balance still only has 3 forces. But now W is increased to W_{t+g}. And because Buoyancy force B does not change, the only possibility for the forces to remain balanced is by reducing Sidewall Friction D. That is possible by decreasing the vertical velocity of the thermal v.

$$v = \sqrt{\frac{rg}{C_d \rho_{air}} \left(\rho_{air} - \rho_{t+g}\right)} = \sqrt{\frac{rg}{C_d \rho_{air}} \left(\rho_{air} - \rho_{thermal} - \frac{m_{gliders}}{\pi\, r^2\, h}\right)}$$

Sidenote: Gliders move slower upwards than thermal through own sink, but they do that by exercising a force upon the air. That force equals the Weight of the glider, and in the simplified model that force is only exercised upon the air in the cylinder of the thermal. That’s convenient in the above derivation.

To obtain the climb rate of the glider, of course we do need to subtract the sinkrate of the glider itself (fixed at 0.8m/s in this model).

So, in the end, the model tells us, the change in vertical velocity of the thermal depends on:

- Relative density difference between air and thermal (without gliders), and thus the temperature of the thermal
- Mass of the gliders
- Radius of the thermal
- Height of the thermal (* in reality probably much less influence)

A major weakness of the model, is that the density distribution of the gliders does not matter. In reality, the gaggle will not be spread out over the whole thermal, but concentrated around a certain altitude.

In reality, in such a situation the climb rates will not be affected the same at all height levels. If you would enter a thermal at 400m below a large gaggle, you be hardly affected at all. Oppositely, at the height level of that large gaggle, the climb rates will reduce more than this model predicts. To find out those effects, we would need a way more complex model.