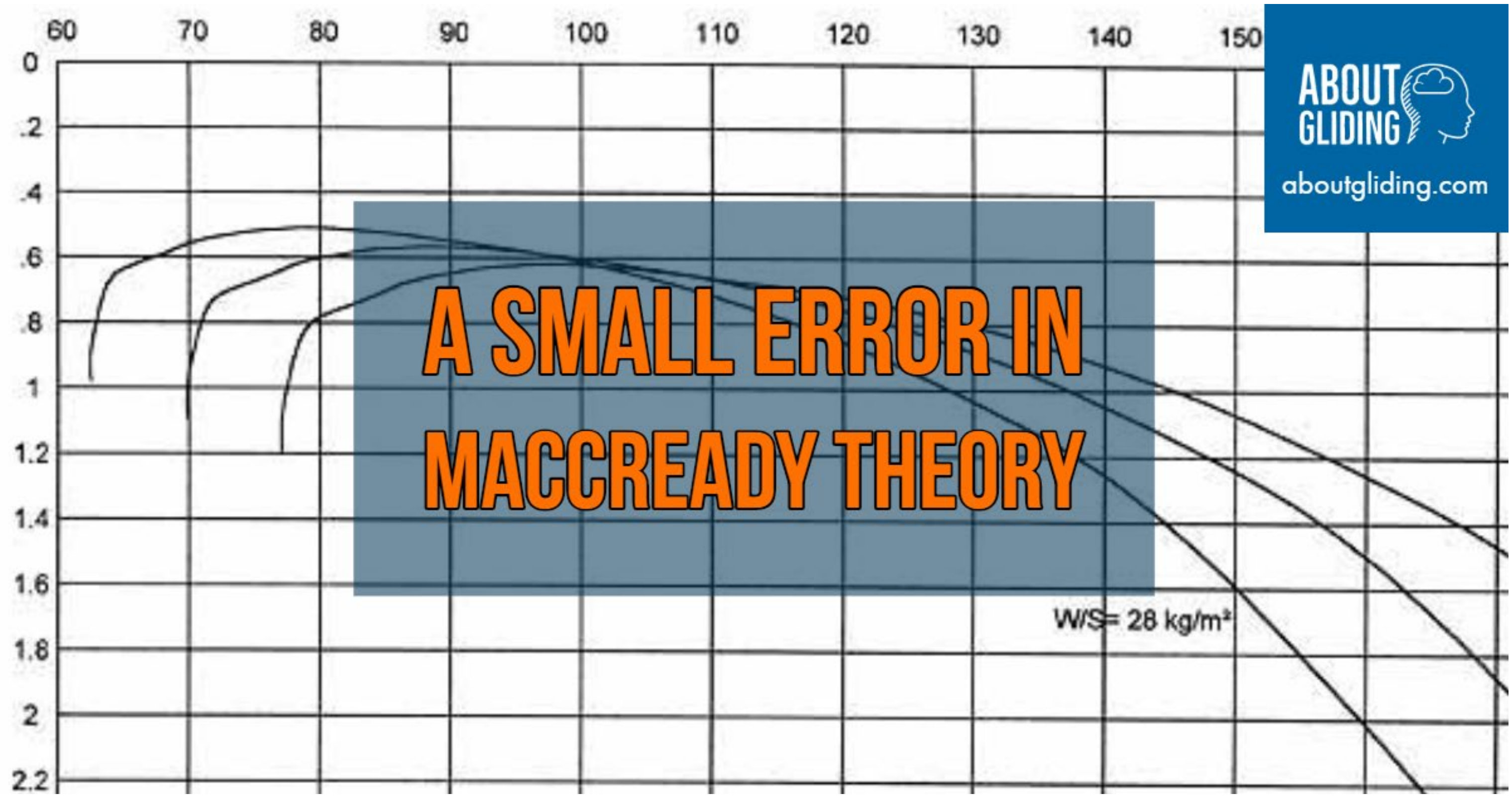

005. A SMALL ERROR IN MACCREADY THEORY.

MC Value setting in the presence of thermal centering losses.

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. Introduction

The crux of 2D MacCready Theory is choosing the right MC Value (MC). The theory then combines the MC Value with the glide polar of your glider and with the horizontal and vertical airmass movement (wind and sinking/rising air during the glide). It then calculates the perfect Speed To Fly (STF). This STF maximizes the average cross country speed.

In flight, the glider pilot must thus simply choose the correct MC Value, and the flight computer provides the correct flight speed. This reduces the workload in the cockpit enormously.

This paper takes one step back and introduces a correction to the common theory of the theoretical correct MC Value, more specifically pertaining to the unavoidable time loss by centering a climb.

While working on another MacCready problem in 2016 and redoing the basics like I had done many times before, I noticed something was off. There seemed to be an error in the math: in the presence of centering losses, the common way of calculating MC settings is wrong. The influence isn't spectacular (1-1.5% in XC speed difference), but interesting nevertheless.

I went back to standard textbooks and articles, to see how different people solved it. Peter Selinger also kindly opened his library and correspondence, so I could investigate the foundational papers as well. To be clear, I haven't read through every text ever published on the matter. But of the many I did read (from the Polish and German original articles in the 1930s, to Nickel and MacCready in the 40s and 50s, over Reichmann's dissertation in the 70s to Cochrane in this millennium, and also about a dozen textbooks), some either gloss over the matter, and most are explicitly incorrect. I couldn't find one which explicitly uses the right approach.

This paper gives a phenomenological explanation, calculated numeric examples, and mathematical proof of the claim. The paper concludes with a section of practical advice in how to implement this in your day to day flying (again, I don't want to oversell it: the influence is relatively limited).

To keep things to the point, I will mainly talk about the simplest case. It remains for certain true for some additions to MacCready theory: wind, sinking/rising airmasses during cruise, inhomogeneous climbs (this last one for all realistic cases); Effects on the latest additions to MacCready theory (course deviations, probability distributions, uncertainty, and limited altitude) haven't been fully researched yet.

Correct MC Value in a homogenous climb with no centering losses

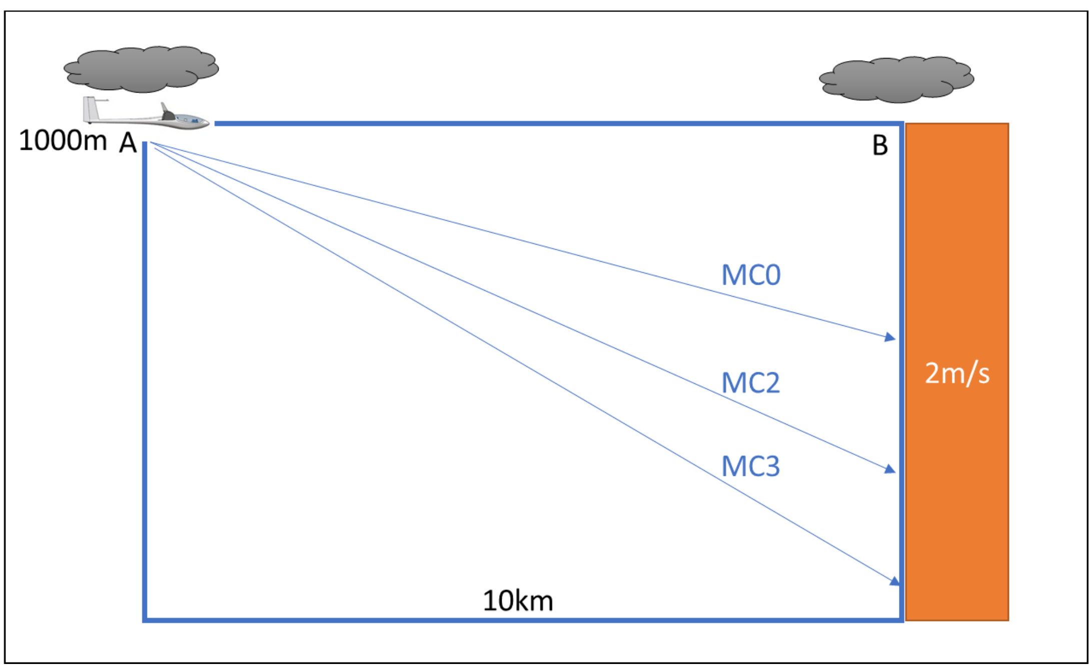
As an introduction, this base case is very simple. The correct answer to this has been known since the 1940s.

Question: The glider is at point A. What is the fastest way to get to point B?

That's what MacCready theory tries to answer.

Assumptions for this example:

- Ventus 2cx-18m @50kg/m2 (polar coordinates a= -0.000088487, b= 0.015641 c= -1.2537)
- Start and finish altitude both at 1000m
- 10km distance
- Climb rate w_{CL} = 2m/s homogenous climb over whole altitude range (this is the climb rate in the thermal which includes own sink speed of the glider), no centering losses
- No airmass movement (no wind, no sinking/rising airmass)



Answer:

- Set MC Value at climb rate of next climb $MC = w_{CL}$ = 2m/s
- Fly correct STF from MC Theory: $\sqrt{\frac{c-w_{CL}}{a}}$ = 191.8km/h with polar coordinates c and a.[1]
- Glide time over 10km is 188sec.
- Glide ratio during cruise is 35.3, altitude loss over the distance of 10km is 283m
- Climb over 283m back to cloudbase takes 142sec.
- Total time is 330sec, and thus the average speed is 109.3km/h.

So the solution to this straightforward base case is simple: **MC setting should be equal to the climb rate of the next climb, and any deviation from the perfect STF will lead to a reduction of average speed over the task.**

Correct MC Value in a homogenous climb with centering losses

The base case can be made more realistic by introducing centering losses. A Table such as below is included in many textbooks and articles about gliding theory. If you need about 60 sec, or about 2 turns, to center a thermal (in this case with 0m/s during the whole time), the total average climb rate is reduced significantly for short and medium climbs (small altitude gain) and high thermal climb rates. It shows clearly how important it is to center your thermals quickly.

60	sec centering at 0m/s					
	ClimbRate m/s					
m alt gain	0.5	1	1.5	2	2.5	3
100	0.38	0.63	0.79	0.91	1.00	1.07
200	0.43	0.77	1.03	1.25	1.43	1.58
300	0.45	0.83	1.15	1.43	1.67	1.88
400	0.47	0.87	1.22	1.54	1.82	2.07
500	0.47	0.89	1.27	1.61	1.92	2.21
600	0.48	0.91	1.30	1.67	2.00	2.31
700	0.48	0.92	1.33	1.71	2.06	2.39
800	0.48	0.93	1.35	1.74	2.11	2.45
900	0.48	0.94	1.36	1.76	2.14	2.50
1000	0.49	0.94	1.38	1.79	2.17	2.54

[1] Derivation and explanation of formulas can be found in the appendix, and are based on the most commonly taught calculations as introduced in Reichmann's book Streckensegelflug

Now, including centering losses, is the perfect speed to fly different from the base case in the previous paragraph?

The text books and theoretical articles I could find say[2]: take the average over the whole climb w_{CL_TOTAL}, ie. take the total height gain and divide by the total time in the climb. That's the correct MC Value. [3]

This is also how I have learned it and believed it to be true for many years, without questioning it.

To calculate this, one would use a formula like this (for centering loss time T_{center} at 0m/s):

$$MC = w_{CL\_TOTAL} = \frac{h_{climb}}{T_{center} + {h_{climb}}/{w_{CL}}}$$

Climb rate w_{CL_TOTAL} depends on the height gained in the climb h_{climb}, and h_{climb} depends on altitude of arrival, which depends on the GlideRatio, which depends on Speed Flown, and thus on the chosen MC Value. This is thus an iterative function.

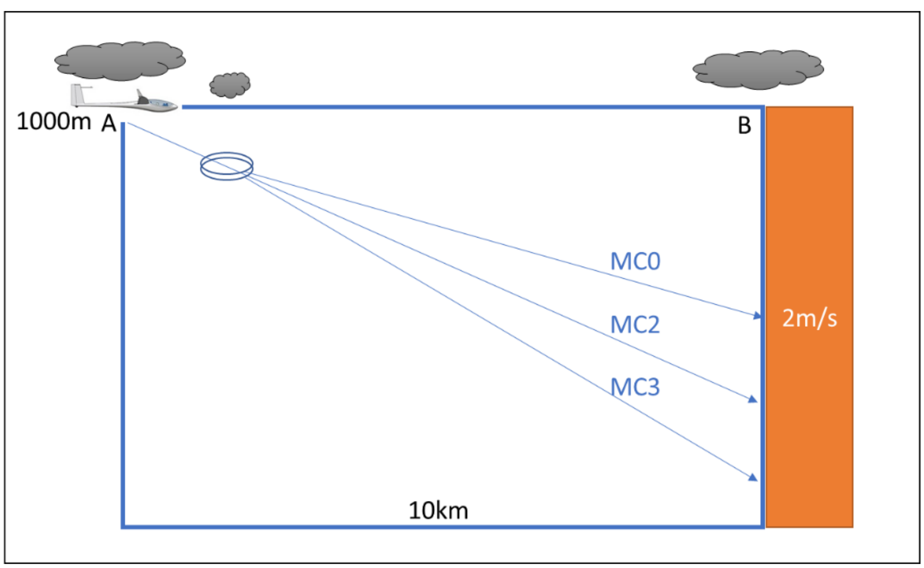
However, the above theory is incorrect.

Phenomenological explanation

Assume the same example as before in the homogenous case without any centering losses. But now after a short distance in the glide, a small suddenly appearing cloud gives a clue of a potential thermal. You make 2 turns under it, without finding any decent thermal. These 2 turns took you 60sec. You decide to continue to the big cloud, as you originally planned to.

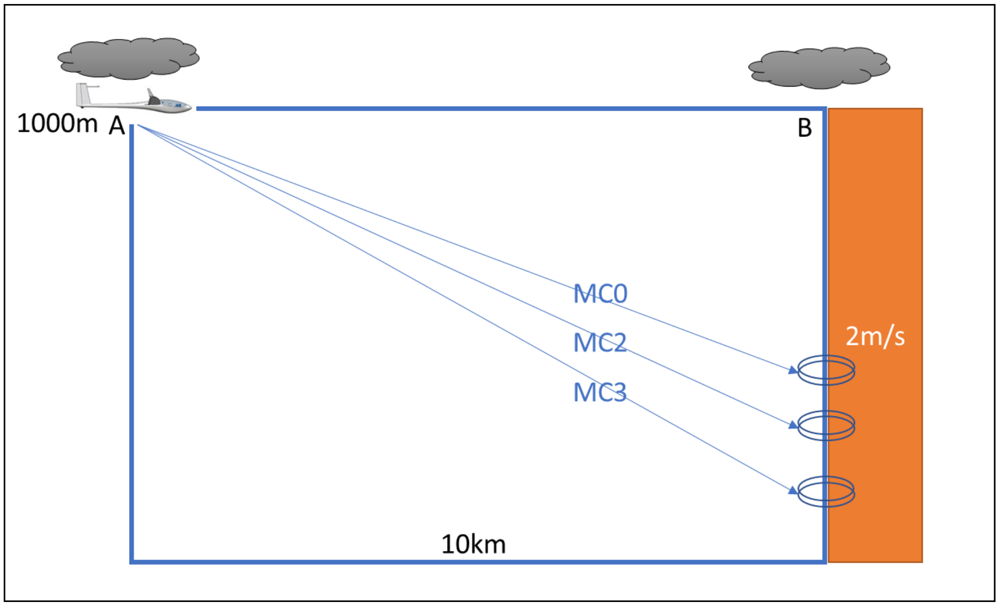
Should these 2 lost turns influence your MC setting, and thus Speed To Fly during any part of the cruise?

No. The optimal case is still flying with MC equal to 2m/s. The 60 seconds are lost in any case, and changing flight speed does not alter that.



[2] Except for the earliest 1930s articles by Szuklowicz and Szwarc, Kasprzyk, and Späte. Those all say to just use the "variometer reading". Of course none at that time made any notice of centering losses. This simplification is normal, as they were just at the beginning of invention the whole theory.
[3] Non-homogenous climbs with different climbrates at different altitude will be addressed later.

If we move those two "lost" turns to the base of the thermal B, being equivalent to centering losses, the same remains true: The best Speed To Fly is still the MC setting equal to the climb rate of the thermal without the centering losses.



The time block (of for instance 60 sec at 0m/s), is independent, of arrival altitude, climb rate, airmass movement during glide and any other value. You will lose this 1 minute of centering no matter what you do. It should thus simply be ignored. The caveat is that centering time and centering climbrate should be independent of arrival altitude.

The correct MC Value is the core climb rate, ignoring all centering time losses!

By core climb rate, I mean the climb rate as it would be if you would immediately center the thermal perfectly. In the homogenous case, that's the given climb rate of the whole thermal w_{CL}. That means the correct MC Value is higher than the total average climb rate over the whole climb, w_{CL_TOTAL}.

A straightforward mathematical proof is given in the appendix.

Making a calculation of the example above, confirms the theory as well:

Assumptions:

- Same as first example above.
- Centering for 60 sec at 0m/s

Table of solutions

							STF for MC =1.345					STF for MC =2				
V Cruise	km./h	150	155	160	165	170	171.4	175	180	185	190	191.8	195	200	205	210
AVG speed	km/h	88.27	89.29	90.15	90.86	91.43	91.56	91.87	92.18	92.38	92.46	92.47	92.45	92.34	92.15	91.87

The table shows that the setting of MC = 2m/s leads to the fastest average speed. Flying at the STF of the iteratively obtained, up till now thought as "perfect", MC Value of 1.345 leads to a 1% loss of average XC speed.

So, in this simple example, we are shown that we should cruise 20km/h faster than common MacCready theory predicts.

Climb rate during centering

The example is with a centering climb rate w_{CE} of 0m/s, which leads to a height gain during centering h_{CE} = 0m. That's a simplification to make the concept clear. But, the mathematics show, that the value of nor w_{CE} , nor h_{CE} matter, as long as the centering climb rate and centering time is independent of arrival altitude. Climb rate during centering w_{CE} can be:

- fixed (eg. 0.6m/s),
- negative (eg. -0.3m/s)
- percentage based (eg. 40% of Thermal climb rate).

This has no influence on the perfect solution for a homogenous thermal.

Also, your MC setting should thus not be influenced by how badly you center climbs.

Useless turns during cruise

If it was not clear from the text above: the same mathematics can be used to prove that useless/lost turns on your way to a thermal should be completely ignored for the correct MC setting.

Inhomogeneous climbs (Climb rate varying with altitude)

For inhomogeneous climbs, w_{CL} and w_{CE} (and thus h_{CE}) can depend on altitude even for an equal centering time. However, it can be proven that in the vast majority of all circumstances the best solution is still the STF/MC chain-rule (Comte/Reichmann), but also ignoring centering losses. And in the small minority of edge cases where it is not correct, the influence of imperfection is negligible.

What if centering time does depend on arrival altitude?

When you get very low, it takes very often a lot longer to center a thermal. That's partly explained by more chaotic thermals down low, and partly by nervousness to land out and thus not flying optimally.

So, in technical terms that means centering time and climb rate do depend on arrival altitude. In that case, the math gets much more complex and depends on de precise circumstances. In general, one could advice to reduce MC settings to not get too low in that bad zone.

Is this useful in praxis?

The exact gain on average speed varies widely with the exact circumstances: time to center, climb rate during centering, type of glider, wing loading, airmass movements, distance between thermals, and of course the core climb rate.

Just 2 additional realistic examples for the Ventus2cxt at 50kg/m2:

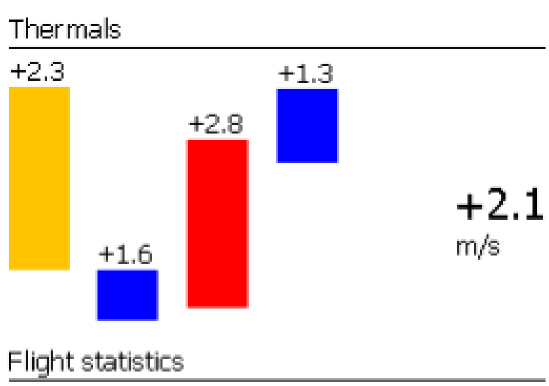
1. Coreclimbrate 3m/s, 0.5m/s cloudstreeting in cruise, 1minute centering at 1m/s, 15km
    i. This is a very good situation.
    ii. oldMC=1.95m/s, oldSTF=174.9km/h, oldXC=127.5km/h
    iii. newMC=3m/s, newSTF=206km/h, newXC=130.25km/h
    iv. -2.08% loss between old and new.

2. Coreclimbrate 1.5m/s, 0.0m/s cloudstreeting in cruise, 1minute centering at 0.5m/s, 8km
    i. This is a not so good situation.
    ii. oldMC=1.14m/s, oldSTF=164.4km/h, oldXC=84.55km/h
    iii. newMC=1.5m/s, newSTF=176.4km/h, newXC=84.93km/h
    iv. -0.44% loss between old and new.

So the difference between previous and new theoretical optimum is around 1-1.5% or so. But that gain does go hand in hand with a bit worse glide ratio, and thus some additional risk.

Now, in praxis we do not know exactly what the next thermals climb rate will be. And thus, uncertainty is added which makes everything more complex.

Often we base our prediction of next climb rates on previous climb rates. A pretty common trick is to use the average climb rate of the 4 last climbs, which you can for instance see on the statistics page of an LX9000 or Oudie (like the +2.1m/s in the picture below).[4]



Depending on how the next clouds look like, you will adapt the number a bit. This number you can read off your screen, does however include centering time losses.

[4] The flight computers of today add up the total altitude gained in the last 4 climbs and divide it by the total time spent in those 4 climbs. The average is thus weighted, and that's the correct way to do this.

Additionally, pilots often choose to fly a bit slower than optimal STF to increase their range and decrease the chance of landing out. Because the optimization curve is rather flat, deviations from the maximum in a rather broad range result in only limited losses of average speed. Pretty typical is to fly ca 15km/h or so below optimal STF.

But here comes the issue: the optimal speed to fly is indeed a pretty broad maximum. However, if you fly already approximately 15km/h slower than the optimal STF because you base yourself on the wrong MC value that includes centering losses (for instance by basing yourself on the flight computers average over the last 4 climbs), and you then choose to subtract another 15km/h to add a bit of glide ratio for more safety, you are now flying 30km/h below the optimal cruisespeed. And thus you are really starting to deviate significantly from the optimal STF. That does start to translate in significant losses.

Take another look at the first example, and its solution table:

							STF for MC =1.345					STF for MC =2				
V Cruise	km./h	150	155	160	165	170	171.4	175	180	185	190	191.8	195	200	205	210
AVG speed	km/h	88.27	89.29	90.15	90.86	91.43	91.56	91.87	92.18	92.38	92.46	92.47	92.45	92.34	92.15	91.87

If you base yourself on MC1.3m/s (the value you would approximately see on the averager of the 4 latest climbs), you would fly 20km/h below optimal, and lose 1% of average XC speed.

If you would see that 1.3m/s on the flight computer and decide to fly an additional 15km/h slower to increase your glide ratio a bit, your XC speed losses would increase to 3.4% against the optimal case. That starts to become noticeable in real life.

So, my tip would be, don’t add an additional margin of safety. Just use the value based on the average of the 4 latest climbs (adapted to the clouds in front of you of course), and don’t fly slower than that. That way, a bit safety is built in, and you still remain in the broad optimal maximum range.

Of course, everything is different when things really start to look hairy and landing out becomes more of a risk. I do regard John Cochrane’s way of viewing at things as correct: the correct MacCready value setting is simple: it is the climb rate you would accept at that given moment. But that’s beyond the scope of this paper.

Appendix: Mathematical proof

Optimal Speed To Fly towards a Homogenous climb (without centering losses)

D distance (independent)
T total time
Tcr cruise time
Tcl climb time
Vcr speed cruise
wcr vertical speed cruise
LD glide ratio in cruise
wcl vertical speed in climb (independent)
hcr altitude loss in cruise
hcl altitude gain in clim
wam vertical airmass movement during cruise (independent)

$$T = Tcr + Tcl$$

$$Tcr = \frac{D}{Vcr}$$

$$Tcl = \frac{hcl}{wcl}$$

$$hcl = -hcr$$

$$hcr = \frac{D}{LD}$$

$$LD = \frac{Vcr}{wcr}$$

$$hcl = -\frac{D\,wcr}{Vcr}$$

Parabolic approximation of the glide polar

$$wcr = a\,Vcr^2 + b\,Vcr + c$$

With a, b and c values dependent on glider performance and air density.

Combining above formulas:

$$T = \frac{D}{Vcr} - \frac{D\ wcr}{Vcr\ wcl}$$

$$T = \frac{D}{Vcr} - \frac{D\ (a\,Vcr^2 + b\,Vcr + c)}{Vcr\ wcl}$$

$$T = \frac{D}{wcl}\ \frac{(wcl - a\,Vcr^2 - b\,Vcr - c)}{Vcr}$$

To find the minimal time to complete the objective, we differentiate the total time to the only variable left: the flight speed Vcr. And then we need to find the roots of the function.

$$\frac{dT}{dVcr} = \frac{d}{dVcr}\left(\frac{D}{wcl} \quad \frac{(wcl - \; a\,Vcr^2 - b\,Vcr - c)}{Vcr}\right)$$

$$\frac{dT}{dVcr} = \frac{-\,D}{wcl} \quad \frac{(a\,Vcr^2 + wcl - c)}{Vcr^2}$$

This function should thus be zero, and the correct root is:

$$Vcr = \sqrt{\frac{c - wcl}{a}}$$

This is the perfect solution for the highest average speed over this task.

Including the vertical airmass movement wam during cruise is straightforward, and leads to the formula:

$$Vcr = \sqrt{\frac{c + wam - wcl}{a}}$$

Optimal Speed To Fly towards a Homogenous climb with centering losses

D distance (independent)
T total time
Tcr cruise time
Tcl climb time
Tce center time (independent)
Vcr speed cruise
wcr vertical speed cruise
LD glide ratio in cruise
wcl vertical speed in climb (independent)
wce vertical speed during centering (independent)
hcr altitude loss in cruise
hcl altitude gain in climb
hce altitude gain during centering

$$T = Tcr + Tcl + Tce$$

$$Tcr = \frac{D}{Vcr}$$

$$Tce = independent = \frac{hce}{wce}, \quad with \lim_{\mathrm{wce}\to 0}\left(\frac{\mathrm{hce}}{\mathrm{wce}}\right) = Tce$$

$$Tcl = \frac{hcl}{wcl}$$

$$hcl = -hcr - hce$$

$$hce = Tce.wce$$

$$hcr = \frac{D}{LD}$$

$$LD = \frac{Vcr}{wcr}$$

$$Tcl = \frac{1}{wcl}\left[\frac{-D\,wcr}{Vcr} - hce\right]$$

Parabolic approximation of the glide polar

$$wcr = a\,Vcr^2 + b\,Vcr + c$$

With a, b and c values dependent on glider performance and air density.

Combining above formulas:

$$T = \frac{D}{Vcr} + \frac{hce}{wce} + \frac{1}{wcl}\left[\frac{-D\,wcr}{Vcr} - hce\right]$$

$$T = \frac{D}{Vcr} - \frac{D\ wcr}{Vcr\ wcl} + \left[\frac{hce}{wce} - \frac{hce}{wcl}\right]$$

$$T = \frac{D}{Vcr} - \frac{D\ (a\,Vcr^2 + b\,Vcr + c)}{Vcr\ wcl} + \left[\frac{hce}{wce} - \frac{hce}{wcl}\right]$$

$$T = \frac{D}{wcl}\ \frac{(wcl - \ a\,Vcr^2 - b\,Vcr - c)}{Vcr} + \left[\frac{hce}{wce} - \frac{hce}{wcl}\right]$$

The term

$$\left[\frac{hce}{wce} - \frac{hce}{wcl}\right]$$

is the only difference with the base case. It becomes equal to 0 if wce = wcl, i.e. no centering losses because climb rate during centering then equals normal climb rate.

The term is equal to a fixed Tce if wce approaches the limit of 0m/s.

To find the minimal time to complete the objective, we differentiate the total time to the only variable left: the flight speed Vcr. And then we need to find the roots of the function.

$$\frac{dT}{dVcr} = \frac{d}{dVcr}\left(\frac{D}{wcl}\ \frac{(wcl - \ a\,Vcr^2 - b\,Vcr - c)}{Vcr} + \left[\frac{hce}{wce} - \frac{hce}{wcl}\right]\right)$$

If Tce, wce, hce and wcl are independent of Vcr (i.e. independent of arrival altitude at the climb), then the derivative of the new term is equal to 0, and the case is simplified to the previous case with the same solution. This is true for homogenous climbs, and if centering time and centering climb rates are independent of arrival altitude.

$$\frac{dT}{dVcr} = \frac{D}{wcl}\ \frac{(a\,Vcr^2 + wcl - c)}{Vcr^2}$$

This function should thus be zero, and the correct root is, exactly the same as before:

$$Vcr = \sqrt{\frac{c - wcl}{a}}$$

Contrarily to commonly taught theory, ***this means that the correct MC Value is independent of time lost during centering the thermal.***